由樂器原理啓發的管道噪音控制

Duct Noise Control Inspired by Musical Instruments

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Abstract: A novel passive duct noise control device is presented. A typical such device consists of an expansion chamber with two side-branch cavities covered by membranes under a fairly high tension, which reacts to incident sound waves like a drum, hence the name drumlike silencer. The first two axial vibration modes of the tensioned membrane are found to be significant in sound reflection. The first mode is effective in generating reflecting sound, but is difficult to excite due to the air stiffness in a compact cavity. The second mode is easier to excite but is less effective in reflecting sound as it is a dipole-like radiator. By setting proper tension on the membranes, the two modes can supplement each other in achieving a broadband silencing performance. The required tension is typically on the same order of magnitude of the tensile strength of typical metallic alloys. To reduce the membrane tension required, the long membrane can be broken into two short pieces and set apart by a segment of solid wall, which leads to a flute-like silencer. Theoretical studies show that both silencers perform well in the low frequency range.

I. Introduction

Duct noise control finds numerous domestic and industrial applications, such as ventilation pipework, vehicle exhaust, etc. Medium and high frequency noise is easily tackled by duct lining using porous materials, for which the technology is rather mature. Low-frequency noise remains a technical challenge. Active control could, in principle, deal with such problem with ease, but the actual application of the technology has been few due to cost, reliability and other practical concerns.

Traditionally, passive control of low frequency noise relies heavily on the structural resonance; membrane and plate have been used in many cases and their main effect has been the mass required to achieve such low frequency resonance, e.g. [1-3]. Recently, however, a different approach has been proposed by our group [4-10], which utilizes the full sound-structure coupling to achieve a broadband control in the low-frequency region. As shown in Fig. 1, the basic prototype resembles an expansion chamber with two side-branch cavities, each covered by a tensioned membrane. When a sound wave is incident from the left-hand side of the device, it induces the membranes to vibrate and this vibration radiates sound waves into both upstream and downstream. The upstream radiation represents sound reflection, while the downstream radiation interferes destructively with the incident wave, leading to noise attenuation in the downstream region. In order to achieve a broad stop-band, the tension required is typically on the same order of magnitude of the tensile strength of typical metallic alloys, hence the name "drumlike". The acoustics of this interference is similar to that of a working rig of active duct noise control. The difference, which is an important one, is that the drumlike silencer is purely reactive and the destructive interference is guaranteed since the incident wave is the sole source of sound power.

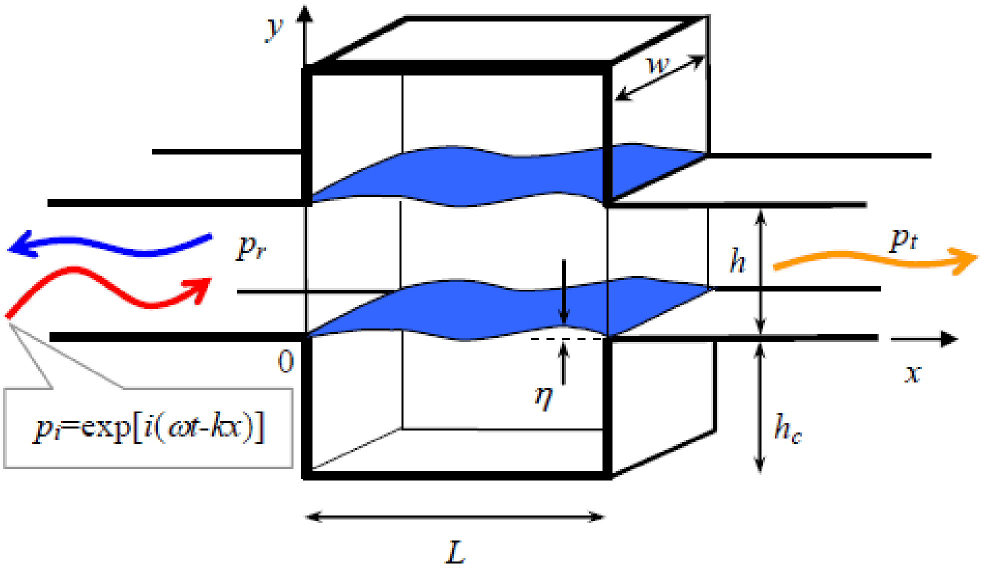


Figure 1. Configuration of a drumlike silencer with two identical modules of cavity- backed membrane. Each cavity has a length of L, depth h_c, and width w.

In what follows, Sec. II briefly describes the theoretical modeling of the membrane response to a grazing incident sound wave. Section III gives the spectrum of transmission loss of a typical drumlike silencer as well as the relationship between the silencing performance and the response of the tensioned membrane. The silencing performance is found to derive mainly from the first two modes of the membrane, which supplements each other in achieving a broadband noise reduction. In Sec. IV, role of the membrane tension is discussed. In an effort to reduce the membrane tension required, the concept of the drumlike silencer is extended to a flute-like silencer, in which the long membrane is broken into two short pieces and set apart by a segment of solid wall.

II. Theoretical Modeling

This section describes the theoretical model for the coupled dynamics of the tensioned membrane and the sound waves. To facilitate the presentation, all variables are normalized by three quantities: fluid density ρ_0^*, speed of sound c_0^*, and duct height h^*. The normalization schemes for some typical parameters are given below

$$x = \frac{x^*}{h^*},\quad y = \frac{y^*}{h^*},\quad \eta = \frac{\eta^*}{h^*},\quad L = \frac{L^*}{h^*},\quad f = \frac{f^* h^*}{c_0^*},\quad m = \frac{m_s^*}{\rho_0^* h^*},\quad T = \frac{T^*}{h^* \rho_0^* \left(c_0^*\right)^2} \quad (1)$$

where f is the dimensionless frequency, m is the mass ratio, and T is the dimensionless tensile force. As shown in Fig. 1, a rectangular duct of height h and width w is equipped with two rigid-walled cavities of length L and depth hc on two opposite sides. Each cavity is covered by a membrane of length L and width w, which forms part of the otherwise rigid duct wall. The leading and trailing edges of the membranes are fixed at x=0, L, while the other lateral edges are set free. Apparently, such a three-dimensional configuration can be reduced to a two-dimensional problem in the x-y coordinate system. Suppose that a plane incident wave comes from the left-hand side of the duct with unit amplitude

$$p_i = \exp\left[i\left(\omega t - k_0 x\right)\right] \qquad (2)$$

and it causes the lower membrane to vibrate with a transverse displacement η(x,t). The dynamics equation for the lower membrane with its nominal position at y=0 is,

$$m\frac{\partial^2\eta}{\partial t^2} - T\frac{\partial^2\eta}{\partial x^2} + \left(p_i + \Delta p\right) = 0, \qquad (3)$$

where pi is the incident wave defined in Eq. (2), Δp=p+rad – p–rad is the fluid loading on the upper (+)and lower (–)sides of the membrane by the membrane vibration itself.

In Eq. (3), the air and membrane motion is strongly coupled via the fluid loading item, Δp. For harmonic vibrations, and introducing membrane velocity V=∂η/∂t=iωη, Eq. (3) becomes,

$$mi\omega V - \left(T/i\omega\right)\partial^2 V/\partial x^2 + \left(p_i + \Delta p\right) = 0. \qquad (4)$$

Equation (4) can be solved via the standard Galerkin procedure, in which V is expanded as a series of in vacuo modes with modal amplitude Vj,

$$V_j\left(t\right) = \frac{2}{L}\int_0^L V\left(x,t\right)\sin\left(j\pi x/L\right)dx, \qquad (5)$$

Once the coupled dynamic Eq. (4) is solved, the membrane radiation wave p+rad into the main duct can be found through modal summation (see Ref. 11). The final transmitted wave pt is then found by adding the incident wave pi to the far-field radiation wave p+rad, and the transmission loss is calculated as (with $|p_i| = 1$),

$$TL = -20\log_{10}\left|p_i\right|. \qquad (6)$$

Similarly, the complex amplitude of the reflected sound pr is the sum of contributions made by all individual membrane vibration modes,

$$p_r = \frac{1}{2}\int_0^L V(x')e^{ik_0x'}dx' = \sum_{j=1}^{\infty} V_j R_j \ , \tag{7}$$

where Rj is the complex amplitude of the reflected sound by the induced vibration of the jth mode with unit amplitude,

$$R_j = \frac{1}{2}\int_0^L \sin(j\pi x'/L)e^{-ik_0x'}dx' = \frac{j\pi L}{2}\exp\left(ik_0L/2\right)\frac{1-e^{i\left(j\pi - k_0L\right)}}{\left(j\pi\right)^2-\left(k_0L\right)^2} \,. \tag{8}$$

III. Performance of the Drumlike Silencer

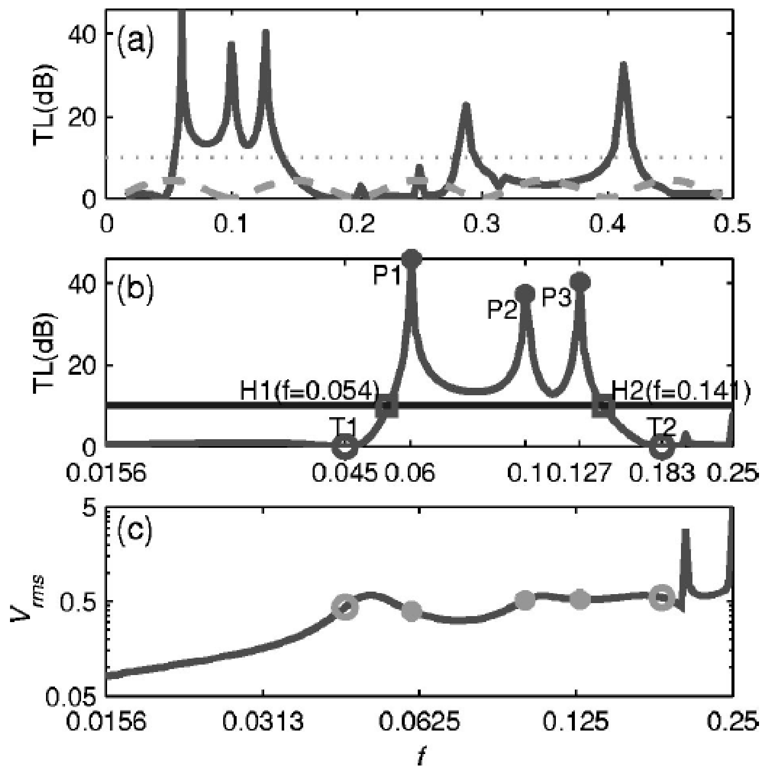
The silencing performance of the drumlike silencer is calculated with the following set of parameters:

$$m=1,\ T=0.475,\ h_c=1,\ L=5. \tag{9}$$

The mass ratio m is considered to be in the practical range as illustrated by the example of 0.077 mm thick aluminum foil used in a duct of height 170 mm, m=2700×0.077/(1.225×170)=1. The dimensionless tensile force T=0.475 translates into a dimensional force of $F^* = T\rho_0^*\left(c_0^*h^*\right)^2 = 1944\ \mathrm{N}$ for a square duct (3D) and the corresponding tensile stress is 148.5 MPa, which is close to but still within the yielding strength of the material.

Figure 2 shows the results for the default configuration specified in (9). The scale of frequency in Fig. 2(a) is linear, and it covers the whole plane wave frequency for the rigid square duct, $f \in \left(0, 0.5\right)$. The horizontal dashed line is the threshold level 10 dB. The dashed curve in Fig. 2(a) is drawn for the plane-wave theory of the expansion chamber of an area ratio of 3. In comparison with the expansion chamber of the same geometry, the drumlike silencer demonstrates much better noise attenuation performance in the low frequency range below f<0.15, which is the range of focus for the present study. Figures 2(b) and 2(c) use a logarithmic scale for the four octave frequency band from f=0.0156 to 0.25. The ranges of frequencies shown in these two subfigures are identical, but in Fig. 2(b) the frequencies of troughs and peaks are labeled for easy reference. Three spectral peaks are marked in both Figs. 2(b) and 2(c) as P1, P2, and P3. Figure 2(c) shows a gradually increasing root-mean-square (RMS) level of the membrane response. One important observation here is that the peaks and troughs in Fig. 2(b) do not correspond to the level of high or low membrane response in Fig.

2(c), as shown by the lack of correlation between the marked peaks and troughs with the actual variation pattern of Vrms. In other words, the effectiveness of the membrane to reflect sound does not solely depend on the amplitude of the induced membrane vibration, but also



on the acoustic interference of sound radiated by different parts of the membrane.

Figure 2. Performance of two opposite membranes under tension T=0.475. The dashed curve in (a) is for a simple expansion chamber with an area ratio of 3.
The peaks are illustrated by **.** and the troughs by◦.

The membrane response is analyzed in terms of the first two in vacuo modes, as shown in Fig. 3. A TL spectrum is attached on top of each column for the purpose of identifying important frequencies. The first vibration mode is very effective in generating sound reflection in the low frequency range, as represented by the high amplitude of reflected sound by the first mode with unit amplitude, $|R_1|$, as shown in Fig. 3(2a). However, as shown in Fig. 3(1a), the first mode is difficult to excite due to the air stiffness in a compact cavity, the compactness being a desirable design attribute. In other words, the cavity volume controls the first mode. On the contrary, the second vibration mode is easier to excite but its capability to cause sound reflection is not as effective as the first mode, c.f. Figs. 3(1b) and 3(2b). The two modes are found to supplement each other in achieving a broadband silencing performance.

The effectiveness of the second mode depends on the ratio of the membrane length to the acoustic wavelength. The second mode has a vibration pattern of $\sin(2\pi x/L)$ along the length of the membrane, cf. Eq.(5), which in fact is a dipole-like radiator. Due to the 180 degree out of phase motion, much of the sound radiated by the leading edge portion of the membrane would be cancelled by that from the trailing edge portion. When the frequency f→0, this self-cancellation would be rather complete since the membrane length L is negligible compared with the acoustic wavelength λ. Therefore, the sound reflection by the second vibration mode becomes significant only when there is noticeable phase difference of sound over the length of the membrane.

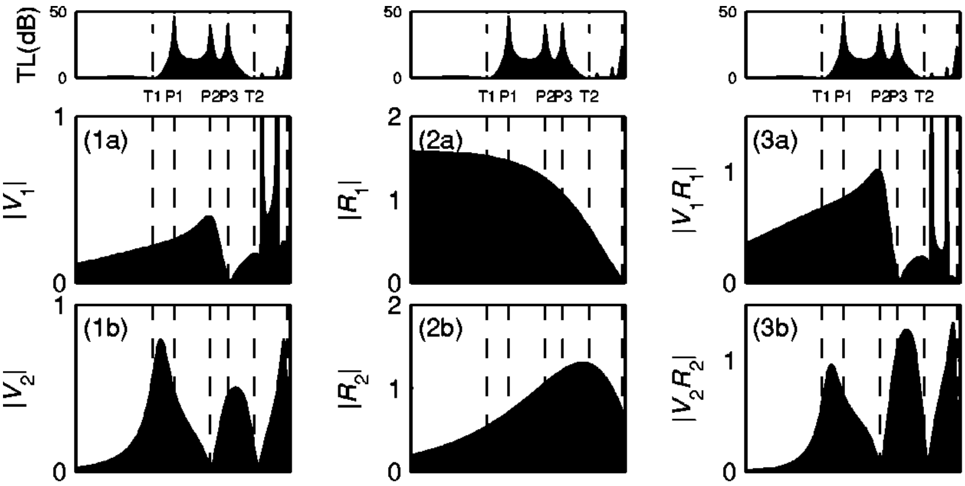


Figure 3. Modal reflections by the first and second in vacuo modes.

IV. Concluding remarks

The tensioned membrane is used to "radiate" effectively so as to cause significant sound reflection. When the membrane is too loose, reflection waves from different parts of the membrane simply cancel themselves out. When the membrane is too tight, it approaches the condition of a hard wall. The mechanism of the superior sound reflection capability achieved for the configuration specified in (9) lies in the delicate acoustic interference between different parts of the tensioned membranes. An optimal tension exists and the value of such tension depends on the choice of the objective function in the optimization process. If the tensioned membrane is replaced by a light but stiff plate with its natural bending moment as the structural restoring force, the resultant plate silencer can also function effectively as a low frequency wave reflector over a very broad frequency band, about one octave band broader than a drumlike silencer of the same cavity geometry [9,10].

For long membranes to respond in low order modes, high tension is required to keep the membrane to vibrate together. It is clear that the required tension would hit the tensile strength of the material if reflection of very low frequency is desired. Aiming for an alternative device for very low frequency noise control without using materials to their limits, the long membrane is broken into two short pieces and set apart by a segment of solid wall, as shown in Fig. 4. This device resembles a flute, hence the name flute-like silencer [12]. Theoretical study shows that flute-like silencer performs well in the low frequency region when compared with the drumlike silencer. However, since the two apertures are much shorter than the membrane in the drumlike silencer, the required membrane tension is much lower than that required in the drumlike silencer. For the aperture design, the parametric study in [12] shows that it is well within the material limits while the plate aperture is still beyond the limits of existing bulk materials.

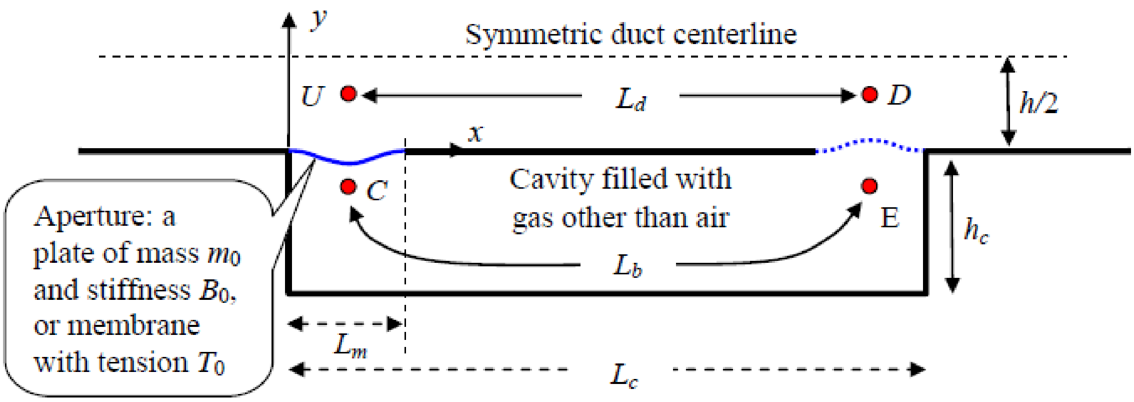


Figure 4. Illustration of a symmetric half of a flute-like silencer.

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